ME 314 - Engineering Design : Mechanical Components
Lecture 24

Load Distribution Factor, K_m

This factor is intended to correct for uneven distribution of the transmitted load Wt over the face width of the gear teeth which can be caused by any misalignment in tooth form.

Suggested values of K_m are shown in Table 11-16. As shown, K_m increases with the face width. A useful rule of thumb is to keep the face width F within the limits

Face Width in	(mm)	K_m
≤ 2	(50)	1.6
6	(150)	1.7
9	(250)	1.8
≥ 20	(500)	2.0

Table 11-16
Load Distribution Factors K_m.

$$8/p_d < F < 16/p_d$$

with nominal values of $12/p_d$. This ratio is referred to as the face width factor.

Size Factor, K_s

This plays the same role as the size factor for correcting the endurance limit in fatigue loading where the test specimen was 0.3 in. Here, many of the available test data are for actual gears and are more precise. As a result, AGMA recommends $K_s = 1$, unless for particular cases, such as very large teeth in which case a value of 1.25 to 1.5 would be a conservative assumption.

Rim Thickness Factor, K_B

Some large-diameter gears are made with a rim and spokes rather than as a solid disk. Depending on how thin the rim depth, t_R, is as compared to the tooth depth, h_t, such designs can fail by radial fracture across the rim rather than through a tooth root. The AGMA defines a backup ratio m_B as

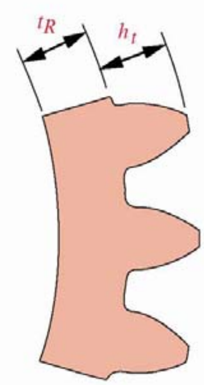


Figure 11-23
Parameters for AGMA Rim Thickness Factor K_B.

$$m_B = t_R / h_t \qquad (12.20a)$$

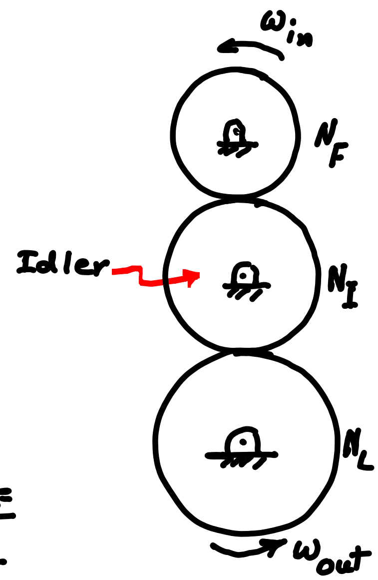
where t_R is measured from the tooth root diameter to the rim's inside diameter and ht is the whole depth of the tooth (the sum of addendum and dedendum). The backup ratio, m_B, is used to define the rim thickness factor as

$$K_B = -2m_B + 3.4 \qquad 0.5 \leq m_B \leq 1.2$$
$$K_B = 1.0 \qquad m_B > 1.2 \qquad (12.20b)$$

$m_B < 0.5$ is not recommended. For solid disks, $K_B = 1.0$.

Idler Factor, K_I

It is common practice to insert an intermediate gear called idler between a pair of gears to make the input and output gears to rotate in the same direction and increase the space between them[a].



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[a]If two shafts that are far apart need to be connected, it will be very expensive to use a "simple" train of many gears (see Sec. 12.5). A chain or bett drive is used for this purpose. Also note that for the simple gear train shown, the velocity ratio is

$$m_V = \left(-\frac{N_F}{N_I}\right)\left(-\frac{N_I}{N_L}\right) = +\frac{N_F}{N_L}$$

Hence, in the case of a simple (series) train, the velocity ratio is always just the ratio of the first gear over the last. Only the sign of the overall train ratio is affected by the idler.

For a pair of gears the ratio is usually limited to about 10:1. To get a ratio of greater than about 10:1 a compound train (in which at least one shaft carries more than one gear) is used. See Section 12-14 of text.

An idler gear is subjected both to more stress cycles per unit time and to greater alternating loads than non-idler gears. To account for this, the idler factor K_I is introduced. We use

$K_I = 1.42$ for idler gears

$K_I = 1.0$ for non-idler gears

Example 3. Bending Stress Analysis of a Spur Gearset

A spur pinion transmits 15 hp at 1200 rpm. It has a pitch of 6 teeth per in, 22 full-depth teeth, and a 20° pressure angle. The gear has 60 teeth. For a face width of 2 in, determine the bending stress in the gear.

Solution:

1. The pitch diameter for pinion is

$$d_p = \frac{N}{P_d} = \frac{22}{6} = 3.67 \text{ in}$$

2. The torque on the pinion shaft is found from Eq. 10.1 :

$$T_p = \frac{P}{\omega_p} = \frac{15 \text{ hp} \left(6600 \ \frac{\text{lb-in}}{\text{s}}/\text{hp}\right)}{1200 \text{ rpm} \left(\frac{2\pi}{60} \ \frac{\text{rad/s}}{\text{rpm}}\right)} = 787.8 \ \text{lb-in}$$

3. The transmitted load is found from the torque & radius :

$$W_t = \frac{T_p}{d_p/2} = \frac{787.8}{3.67/2} = 429.7 \ \text{lb}$$

4. Tooth bending stress is given by Eq 12.15 :

$$\sigma_b = \frac{W_t P_d}{F J} \ \frac{K_a K_m}{K_v} K_s K_B K_I$$

* W_t , P_d , and the face width, F are known.
* The application factor, $K_a = 1$, from Table 12-17, assuming that the source and load are both uniform.
* The load distribution factor, $K_m = 1.6$, from Table 12-16, for a face width of 2 in.
* The size factor, $K_s = 1$ since the teeth are not very large in this case.
* The rim thickness factor, $K_B = 1$ since both the pinion & the gear are too small to have a rim and a spoke.
* The idler factor, $K_I = 1$ for both pinion and gear.
* The velocity factor, K_v is found from Eqs. 12.16 and 12.17, or from Figure 12.22
  The pitch-line velocity, V_t is given by

$$V_t = r_p \omega_p = \frac{d_p}{2} \omega_p =$$

$$=$$

Using Table 12-7 (p.705) for $V_t = 1153$ ft/min, we choose a quality factor, $Q_v = 8$.

$$B =$$

$$A =$$

$$K_v = \left(\frac{A}{A + \sqrt{V_t}} \right)^B =$$

(same result is obtained from Fig. 12-22)

* The bending geometry factor J for the 20°- pressure angle, 22-tooth pinion in mesh with the 60-tooth gear is found from Table 12-9 (p.712), recalling from Example 1 that load is at Highest Point of Single-Tooth Contact (HPSTC). After interpolation, we find

$$J_{pinion} = 0.35 \quad , \quad J_{gear} = 0.40$$

* The pinion-tooth bending stress is

$$\sigma_{b_p} = \frac{W_t P_d}{F J_{pinion}} \frac{K_a K_m}{K_v} K_s K_B K_I$$

$$= \frac{\qquad}{\qquad} \frac{\qquad}{\qquad} (\ )(\ )(\ ) =$$

* The gear-tooth bending stress is

$$\sigma_{b_g} = \frac{W_t P_d}{F J_{gear}} \frac{K_a K_m}{K_v} K_s K_B K_I$$

$$= \frac{\qquad}{\qquad} \frac{\qquad}{\qquad} (\ )(\ )(\ ) =$$

Surface Stresses

As mentioned earlier, the stresses at the tooth surface are dynamic Hertzian contact stresses in combined rolling and sliding. Only at the pitch point the relative motion of gear teeth is pure rolling and the percentage of sliding increases with distance from the pitch point. These 3D stresses have their peak values either on the surface or slightly below it. If sufficient, clean lubricant of an appropriate type is provided to prevent adhesive, abrasive, or corrosive failures, the ultimate failure mode will be pitting and spalling due to surface fatigue.

The surface stresses in gear teeth are established by AGMA on the basis of equations developed by Buckingham as reported in his 1949 book entitled: Analytical Mechanics of Gears. The AGMA's pitting resistance formula is

$$\sigma_c = C_p \sqrt{\frac{W_t}{FId} \frac{C_a C_m}{C_v} C_s C_f} \qquad (12.21)$$

where W_t = transmitted force on the tooth

d = pitch diameter of the smaller of two gears in mesh

F = face width

I = surface geometry factor

C_p = elastic coefficient

C_f = surface finish factor

$C_a = K_a$ application factor)

$C_m = K_m$ (load distribution factor)

$C_V = K_V$ (dynamic factor)

$C_s = K_s$ (size factor)

Surface Geometry Factor, I

AGAMA defines this factor in terms of the radii of curvature of pinion and gear, ρ_p and ρ_g, the pressure angle, φ, and the pinion pitch diameter, d_p, as

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}\right) d_p} \qquad (12.22a)$$

where the upper sign applies to external gearsets in all expressions in this section. The radii of curvature are calculated from

$$\rho_p = \sqrt{\left(r_p + \frac{1+x_p}{p_d}\right)^2 - (r_p \cos\phi)^2} - \frac{\pi}{p_d}\cos\phi \qquad (12.22b)$$

$$\rho_g = C\sin\phi \mp \rho_p$$

where p_d = diametral pitch

r_p = radius of pinion

φ = pressure angle,

C = center distance between pinion and gear

and where x_p = pinion addendum coefficient which equals the percentage of addendum elongation. For standard full-depth teeth, $x_p = 0$. For 25%-long-addendum teeth, $x_p = 0.25$, etc.

Elastic Coefficient, C_p

This accounts for differences in tooth material and is given by

$$C_p = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_p}\right)+\left(\frac{1-\nu_g^2}{E_g}\right)\right]}} \qquad (12.23)$$

where ν_p and ν_g are the Poisson's ratios, and E_p and E_g are the moduli of elasticity for pinion and gear materials. Note that the units of C_p are either, √psi or √MPa.

Surface Finish Factor, C_f

This factor is introduced to account for unusually rough surface finishes on gear teeth. However, AGMA has not yet established standards for it and recommends C_f = 1 for gears made by conventional methods.

Example 4. Surface Stress Analysis of a Spur Gearset

A spur pinion transmits 15 hp at 1200 rpm. It has a pitch of 6 teeth per in, 22 full-depth teeth, and a 20° pressure angle. The gear has 60 teeth. For a face width of 2 in, determine the contact-stress given that the pinion and gear are both made of steel.

Solution:

1. The pitch diameter for pinion is

$$d_p = \frac{N_p}{P_d} = \frac{22}{6} = 3.67 \text{ in} \quad , \quad d_g = \frac{N_g}{P_d} = \frac{60}{6} = 10 \text{ in}$$

2. The torque on the pinion shaft is found from Eq. 10.1:

$$T_p = \frac{P}{\omega_p} = \frac{15 \text{ hp} \left(6600 \ \frac{\text{lb-in}}{\text{s}}/\text{hp}\right)}{1200 \text{ rpm} \left(\frac{2\pi}{60} \ \frac{\text{rad/s}}{\text{rpm}}\right)} = 787.8 \text{ lb-in}$$

3. The transmitted load is found from the torque & radius:

$$W_t = \frac{T_p}{d_p/2} = \frac{787.8}{3.67/2} = 429.7 \text{ lb}$$

4. The tooth-surface stress (AGMA's pitting resistance) formula is:

$$\sigma_c = C_p \sqrt{\frac{W_t}{F I d} \frac{C_a C_m}{C_v} C_s C_f} \qquad (12.21)$$

* W_t, d, and the face width, F are known.

* The application factor, $C_a = K_a = 1$, from Table 12-17, assuming that the source and load are both uniform.

* The load distribution factor, $C_m = K_m = 1.6$, from Table 12-16, for a face width of 2 in.

* The size factor, $C_s = K_s = 1$ since the teeth are not very large in this case.

* The velocity factor, $C_v = K_v$ is found from Eqs. 12.16 and 12.17, or from Figure 12-22 (p.711).

The pitch-line velocity, V_t is given by

$$V_t = r_p \omega_p = \frac{d_p}{2}\omega_p = \left(\frac{3.67}{2}\text{ in}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)(1200\text{ rpm})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)$$
$$= 1153\text{ ft/min}$$

Using Table 12-7 (p. 705), for $V_t = 1153$ ft/min, we choose a quality factor, $Q_v = 8$.

$$B = \frac{(12-8)^{2/3}}{4} = 0.630$$

$$A = 50 + 56(1 - 0.630) = 70.72$$

$$C_v = K_v = \left(\frac{A}{A + \sqrt{V_t}}\right)^B = \left(\frac{70.72}{70.72 + \sqrt{1153}}\right)^{0.630} = 0.78$$

(same result is obtained from Fig. 12-22, p. 711)

* The surface factor $C_f = 1$ for gears made by conventional methods.
* The elastic coefficient C_p is given by Eq. 12.23 (p. 719):

$$C_p = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_p}\right) + \left(\frac{1-\nu_g^2}{E_g}\right)\right]}} = \sqrt{\frac{1}{\pi\left[\left(\frac{\quad}{\quad}\right) + \left(\frac{\quad}{\quad}\right)\right]}}$$
$$= \qquad \sqrt{psi}$$

where, from Table A-1 (p. 988), $E_p = E_g =$
and $\nu_p = \nu_g =$

* The pitting (or surface) geometry factor I is calculated from Eq. 12-22 (p. 711):

$$\rho_p = \sqrt{\left(r_p + \frac{1 + x_p}{p_d}\right)^2 - (r_p \cos\phi)^2} - \frac{\pi}{p_d}\cos\phi \qquad (12.22b),$$

$$= \sqrt{\left(\quad + \frac{\quad}{\quad} \right)^2 - \left(\quad \right)^2} - $$

$$\rho_g = C \sin\phi - \rho_p = (\qquad) \sin 20^\circ - \qquad =$$

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} + \frac{1}{\rho_g} \right) d_p} = \frac{\cos 20^\circ}{\left(\frac{1}{0.524} + \frac{1}{1.81} \right) 3.67} = 0.104$$

* The surface stress is

$$\sigma_c = C_p \sqrt{\frac{W_t}{F I d} \frac{C_a C_m}{C_v} C_s C_f}$$

$$= (\qquad) \sqrt{\frac{\qquad}{(\ )(\qquad)(\quad)} \frac{(\ )(\quad)}{\qquad} (\ )(\ )} =$$

12.9 Gear Materials

Steels, cast irons, and malleable and nodular irons are the most common choices for gears. In highly corrosive environments-such as in marine environments, bronzes are often used. The combination of a bronze gear and a steel pinion has advantages in terms of material compatibility and conformity (see Chapter 7) and this combination is used in non-marine environments as well. The combination of a steel pinion (for strength in the higher stressed member) and a cast iron gear is also often used.

Materials Strength - Test data for fatigue strengths of most gear materials have been compiled by AGMA so we do not need to use all the correction factors introduced in Chapter 6.

AGMA Bending-Fatigue Strengths for Gear Materials

There are still three correction factors that need to be applied to the published AGMA bending-fatigue strength data $S_{fb'}$ in order to obtain the corrected bending-fatigue strength for gears S_{fb} .

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'} \qquad (12.24)$$

where K_s are correction factors which will be considered next.